ECON3111: Behavioural Economics - Advanced Choice Under Risk and Uncertainity
Allais & Ellsberg, Prospect Theory, and Ambiguity

Mesfin Genie, PhD

NBS, University of Newcastle, Australia

Learning objectives (Week 6)

By the end of today, you should be able to:

1. Explain expected utility (EU) as a benchmark and state why the independence (sure-thing) idea matters. *(Outcomes 1, 3)*
2. Diagnose EU violations using the Allais and Ellsberg paradoxes, in words and with simple algebra. *(Outcomes 1, 2)*
3. Describe and compute prospect theory components: reference point, value function, loss aversion, and probability weighting. *(Outcomes 2, 3)*
4. Apply prospect theory to consumer and managerial decisions (insurance, warranties, gambling, pricing). *(Outcomes 2, 4)*
5. Model ambiguity aversion using multiple priors (maxmin) and solve small numerical examples. *(Outcomes 2, 4)*

Session plan

1. Benchmark: EU and the independence/sure-thing logic
2. Allais paradox: the certainty effect and independence failure
3. Probability weighting: why “1%” does not feel like 1%
4. Prospect theory: reference dependence, loss aversion, weighted probabilities
5. Ellsberg paradox: ambiguity aversion and the sure-thing principle
6. Multiple priors: robust choice under ambiguous probabilities
7. Applications, misconceptions, and exam practice

Where we are

Motivation

Risk and uncertainty appear everywhere:

- **Households:** insurance deductibles, extended warranties, lotteries, superannuation choices.
- **Firms:** R&D projects, new product launches, uncertain demand, contract design.
- **Policy:** disaster preparedness, health screening, regulation under uncertain evidence.

EU is a clean benchmark. Behavioural economics asks: **which deviations are systematic, and what do they imply?**

Key distinctions: risk vs ambiguity

Let a random outcome be X.

- **Risk:** the probability distribution of X is known (or treated as known).
- **Ambiguity:** probabilities are not pinned down; there is a set of plausible distributions.

Today focuses on:

- **certainty effects** (special treatment of probability 1),
- **probability weighting** (non-linear perception of probabilities),
- **ambiguity aversion** (dislike of unknown probabilities).

A warm-up question (quick show of hands)

Which feels more attractive?

1. Receive A$ 100 for sure.
2. Receive A$ 120 with probability 0.9 (and A$ 0 otherwise).

Both expected values are:

$$\mathbb{E}[\text{sure}] = 100, \qquad \mathbb{E}[\text{risky}] = 0.9 \cdot 120 = 108.$$

If many people choose the sure option, EU can explain it with risk aversion, but not all patterns can be explained that way.

Where we are

Lotteries and expected utility (EU)

A lottery L has outcomes $x_1, \ldots, x_n \in \mathbb{R}$ with probabilities $p_1, \ldots, p_n$:

$$p_i \geq 0, \qquad \sum_{i=1}^{n} p_i = 1.$$

Let $u : \mathbb{R} \to \mathbb{R}$ be a utility function over money. EU of L is:

$$\mathrm{EU}(L) = \sum_{i=1}^{n} p_i \, u(x_i).$$

EU prediction: prefer L to M if $\mathrm{EU}(L) > \mathrm{EU}(M)$.

Risk attitudes in EU

If u is twice differentiable:

- risk averse if $u''(x) < 0$ (concave),
- risk neutral if $u''(x) = 0$ (linear),
- risk seeking if $u''(x) > 0$ (convex).

A useful measure: **certainty equivalent (CE)**. For a lottery L, the CE is the sure amount $\mathrm{CE}(L)$ solving:

$$u(\mathrm{CE}(L)) = \mathrm{EU}(L).$$

Risk premium is $\mathbb{E}[X] - \mathrm{CE}(L)$ (how much expected value is given up for certainty).

Worked example 1: CE under EU (AUD)

Lottery: $X = \text{A\$}\,100$ with probability 0.5 and $\text{A\$}\,0$ with probability 0.5. Let $u(x) = \sqrt{x}$ for $x \geq 0$.

Compute:

$$\mathrm{EU} = 0.5\sqrt{100} + 0.5\sqrt{0} = 0.5 \cdot 10 = 5.$$

CE solves $\sqrt{\mathrm{CE}} = 5$, so $\mathrm{CE} = 25$.

Expected value is $\mathbb{E}[X] = 50$, so risk premium is $50 - 25 = 25$ (in dollars).

Independence (sure-thing principle): what it says

Independence (mixing) axiom: If $A \succ B$, then for any C and any $0 < \alpha < 1$,

$$\alpha A + (1-\alpha)C \succ \alpha B + (1-\alpha)C.$$

Interpretation: if two options share the same consequences in some states, those shared parts should not flip preferences. This is closely related to the **sure-thing principle**.

Why independence is plausible (simple EU logic)

Suppose two options differ only in one state, and are identical in the others. Under EU, identical states contribute the same utility to both options, so they cancel in comparison.
This is the benchmark logic that will be violated in the Allais and Ellsberg paradoxes.

Where we are

Allais paradox: the classic choice pattern

Two decisions:

Decision 1

- (1A) A$ 1,000,000 for sure.
- (1B) 89% of A$ 1,000,000, 10% of A$ 5,000,000, 1% of A$ 0.

Decision 2

- (2A) 11% of A$ 1,000,000, otherwise A$ 0.
- (2B) 10% of A$ 5,000,000, otherwise A$ 0.

Common pattern: choose (1A) and (2B).

Allais shown as shared states (table)

	State 1 (0.89)	State 2 (0.10)	State 3 (0.01)
(1A)	1M	1M	1M
(1B)	1M	5M	0
(2A)	0	1M	1M
(2B)	0	5M	0

In State 1, Decision 1 has the same payoff in both options (**a sure component**). Independence says removing a common component should not reverse preferences.

Why (1A) & (2B) violates EU (short algebra)

Normalise only by assuming $u(5\text{M}) > u(1\text{M}) > u(0)$.
Preference (1A)$\succ$(1B) implies:

$$u(1\text{M}) > 0.89u(1\text{M}) + 0.10u(5\text{M}) + 0.01u(0).$$

Preference (2B)$\succ$(2A) implies:

$$0.10u(5\text{M}) + 0.90u(0) > 0.11u(1\text{M}) + 0.89u(0).$$

Rearranging the second inequality gives:

$$0.89u(1\text{M}) + 0.10u(5\text{M}) + 0.01u(0) > u(1\text{M}),$$

contradicting the first. Hence EU cannot generate the pattern.

Intuition: what the Allais paradox is saying

Many people treat:

"probability 1" as categorically different from "probability 0.99".

This is the **certainty effect**. Even small reductions from certainty can feel like a large loss in security.

Worked example 2: certainty effect with small numbers (AUD)

Decision A vs B:

- A: A$ 300 for sure.
- B: 80% chance of A$ 400, otherwise A$ 0.

Expected values: $\mathbb{E}[A] = 300$, $\mathbb{E}[B] = 320$.
Now "scale down":

- C: 25% chance of A$ 300, otherwise A$ 0.
- D: 20% chance of A$ 400, otherwise A$ 0.

Expected values: $\mathbb{E}[C] = 75$, $\mathbb{E}[D] = 80$.
Pattern often observed: choose A over B (certainty), but choose D over C (no certainty). This is a typical EU violation.

Allais as a common consequence effect

Independence says: if two options share a common outcome with some probability, removing that common "consequence" should not change preferences.
Allais-style choices suggest:

- people care about the presence of a **sure** outcome,
- not only about payoffs and probabilities in a linear way.

This motivates probability weighting models.

Integrated learning activity 1: Allais reasoning

Activity (pairs, 3 minutes).

1. Choose (1A) or (1B), and choose (2A) or (2B).
2. Write one sentence explaining why certainty matters to you (if it does).
3. Identify the "shared" part of Decision 1 and explain why EU says it should not matter.

Goal: practise translating an axiom into plain language.

Where we are

From EU to probability weighting

EU treats probabilities linearly: probability p multiplies utility.
A behavioural alternative: people transform probabilities using a weighting function

$$\pi : [0, 1] \to [0, 1], \quad \pi(0) = 0, \ \pi(1) = 1.$$

Interpretation: $\pi(p)$ is **decision weight**, not an objective probability.

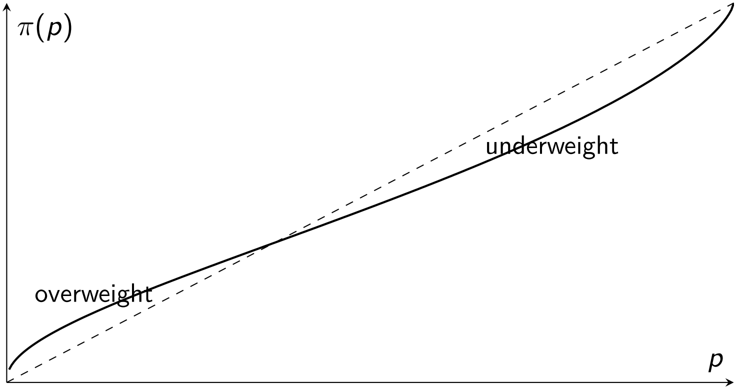
Typical shape: inverse-S weighting

Empirical regularities often described as:

- overweight small probabilities: $\pi(p) > p$ for small p,
- underweight moderate to large probabilities: $\pi(p) < p$ for many p away from 0,
- special sensitivity near certainty: $\pi(1) - \pi(0.99)$ can feel large.

This helps explain lotteries (small p) and insurance (small p of loss) and certainty effects.

Visual: probability weighting



Dashed: $\pi(p) = p$ (EU). Solid: illustration of non-linear decision weights.

Worked example 3: overweighting a small probability

Suppose $p = 0.01$ but it is weighted as $\pi(0.01) = 0.05$.
Compare two gambles:

- Lottery L: win A$ 2,000 with probability 0.01, otherwise A$ 0.
- Sure S: A$ 30 for sure.

Expected values: $\mathbb{E}[L] = 20$, $\mathbb{E}[S] = 30$.
Using weighted value with a linear value function $v(x) = x$ (for illustration):

$$V(L) = \pi(0.01) \cdot 2000 = 0.05 \cdot 2000 = 100, \qquad V(S) = 30.$$

Lottery becomes attractive even though its expected value is lower.

Worked example 4: underweighting a high probability

Suppose $p = 0.9$ but it is weighted as $\pi(0.9) = 0.8$.
Choose between:

- A: A$ 100 for sure.
- B: A$ 130 with probability 0.9, otherwise A$ 0.

Expected values: $\mathbb{E}[A] = 100$, $\mathbb{E}[B] = 117$.
With $v(x) = x$ and weighting:

$$V(A) = 100, \qquad V(B) = \pi(0.9) \cdot 130 = 0.8 \cdot 130 = 104.$$

A small perceived drop in reliability makes the gamble much less attractive than expected value suggests.

Where we are

Prospect theory: three building blocks

Prospect theory explains systematic EU violations using:

1. **Reference dependence:** outcomes evaluated as gains/losses relative to a reference point r.
2. **Loss aversion:** losses hurt more than equal-sized gains feel good.
3. **Probability weighting:** decision weights $\pi(p)$ replace objective p.

Together these produce patterns like:

- certainty effects,
- preference reversals when probabilities change,
- risk aversion for gains but risk seeking for losses (often),
- simultaneous gambling and insurance purchase.

Reference point and changes

Let r be a reference point (status quo, expectation, target). An outcome x is experienced as a change:

$$\Delta = x - r.$$

- If $\Delta > 0$, it is a **gain**.
- If $\Delta < 0$, it is a **loss**.

Prospect theory uses a **value function** $v(\Delta)$ rather than a utility function over total wealth.

Value function: shape and loss aversion

A standard value function satisfies:

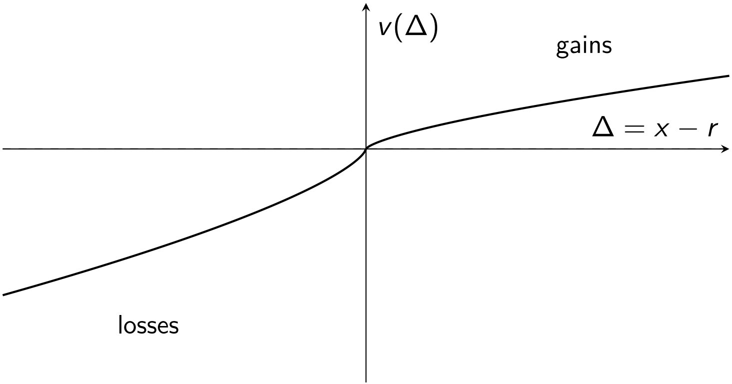
- $v(0) = 0$ (reference dependence),
- diminishing sensitivity: concave for gains, convex for losses,
- loss aversion: $-v(-\Delta) > v(\Delta)$ for $\Delta > 0$.

One convenient parametric form:

$$v(\Delta) = \begin{cases} \Delta^{\alpha}, & \Delta \geq 0, \\ -\lambda(-\Delta)^{\beta}, & \Delta < 0, \end{cases}$$

with $\alpha, \beta \in (0, 1)$ and $\lambda > 1$.

Visual: S-shaped value function



Concave for gains, convex for losses, steeper for losses ($\lambda > 1$).

Worked example 5: loss aversion in numbers (AUD)

Let $\alpha = \beta = 0.5$ and $\lambda = 2$. Then:

$$v(100) = 100^{0.5} = 10, \qquad v(-100) = -2 \cdot (100)^{0.5} = -20.$$

Interpretation: losing A\$ 100 feels about twice as bad (in value units) as gaining A\$ 100 feels good.

Probability weighting inside prospect theory

Prospect theory evaluates risky prospects using decision weights.
For a two-outcome gain prospect: win $\Delta_H > 0$ with probability p, else $\Delta_L = 0$, a simple teaching version is:

$$V = \pi(p)\, v(\Delta_H) + (1 - \pi(p))\, v(0).$$

If $v(0) = 0$, this simplifies to $V = \pi(p)\, v(\Delta_H)$.
Key idea: **small p can be overweighted** and **near certainty can be special**.

Cumulative prospect theory (idea, not memorisation)

For lotteries with multiple outcomes, prospect theory often uses **cumulative** decision weights. Idea:

- Order outcomes from worst to best.
- Weight changes in cumulative probabilities, not each probability separately.

This avoids some technical problems and fits data better. For this course, what matters most is:

1. value over gains/losses relative to r,
2. loss aversion,
3. probability weighting.

Worked example 6: certainty effect via weighting (AUD)

Compare:

- A: sure gain A$ 100.
- B: gain A$ 120 with probability 0.95, else A$ 0.

Let $v(\Delta) = \sqrt{\Delta}$ for gains, and suppose $\pi(0.95) = 0.85$ (underweighting high probability).
Compute:

$$V(A) = \sqrt{100} = 10.$$

$$V(B) = \pi(0.95)\sqrt{120} \approx 0.85 \cdot 10.95 \approx 9.31.$$

Prospect theory predicts the sure option even though 0.95 is high and expected value is 114.

Worked example 7: lottery ticket demand (AUD)

Option S: sure gain A$ 20.
Option L: gain A$ 2,000 with probability 0.01, else A$ 0.
Let $v(\Delta) = \sqrt{\Delta}$ and $\pi(0.01) = 0.05$.
Compute:

$$V(S) = \sqrt{20} \approx 4.47, \qquad V(L) = 0.05 \cdot \sqrt{2000} \approx 0.05 \cdot 44.72 \approx 2.24.$$

Here, sure money wins.
If the prize is A$ 10,000 instead:

$$V(L) = 0.05 \cdot \sqrt{10{,}000} = 0.05 \cdot 100 = 5,$$

making the lottery attractive. Small-probability weighting can generate lottery-like behaviour.

Worked example 8: why warranties sell (AUD)

A laptop repair costs A$ 1,200 with probability $p = 0.04$. A warranty costs A$ 180 and eliminates the repair cost.

Assume:

- reference point is "no repair cost" so the repair is a loss,
- $v(\Delta) = -\lambda\sqrt{-\Delta}$ for losses, $\lambda = 2$,
- $\pi(0.04) = 0.10$ (overweighting a small probability).

No warranty:

$$V_{\text{no}} = \pi(0.04)\, v(-1200) = 0.10 \cdot (-2\sqrt{1200}) \approx 0.10 \cdot (-2 \cdot 34.64) = -6.93.$$

Warranty:

$$V_{\text{w}} = v(-180) = -2\sqrt{180} \approx -26.83.$$

This parametrisation predicts no warranty. If λ is larger, or if $\pi(0.04)$ is larger, or if the repair is framed as a severe loss, the warranty can become preferred.

What to learn from the warranty example

The purpose is not one “correct” number. It is to see how three ingredients shift behaviour:

- **reference point:** is the repair framed as a loss?
- **loss aversion:** how painful is a loss relative to a gain?
- **probability weighting:** does a 4% risk feel like 4%?

Different consumers (and different frames) can produce different choices.

Integrated learning activity 2: prospect theory calculation

Activity (individual, 4 minutes). Use the simplified evaluation.
Let $v(\Delta) = \sqrt{\Delta}$ for gains and $v(0) = 0$. Suppose $\pi(0.10) = 0.18$ and $\pi(0.90) = 0.80$.
Compute the prospect values:

1. A: sure gain A$ 90.
2. B: A$ 1,000 with probability 0.10, else A$ 0.
3. C: A$ 120 with probability 0.90, else A$ 0.

Rank A, B, C from most to least attractive.

Where we are

1. Motivation and real-world relevance
2. Benchmark: expected utility and the independence idea
3. Allais paradox: the certainty effect
4. Probability weighting: why probabilities may not feel linear
5. Prospect theory: mechanics and intuition
6. **Ellsberg paradox: ambiguity aversion**
7. Multiple priors: a model of ambiguity aversion
8. Applications: consumer, business, and policy decisions
9. Common misconceptions and pitfalls
10. Exam preparation
11. Key takeaways and next week

Ellsberg setup: known vs unknown probabilities

An urn has 90 balls:

- 30 red (known),
- 60 black or yellow in an unknown split.

A ball is drawn at random.
Decision 1:

- (I) win A$ 100 if red.
- (II) win A$ 100 if black.

Decision 2:

- (III) win A$ 100 if red or yellow.
- (IV) win A$ 100 if black or yellow.

Common pattern: choose (I) and (IV).

Ellsberg payoff table

	Red (R)	Black (B)	Yellow (Y)
(I)	100	0	0
(II)	0	100	0
(III)	100	0	100
(IV)	0	100	100

Within each decision, the Yellow payoff is the same across the two options:

- In Decision 1, both give 0 in Y.
- In Decision 2, both give 100 in Y.

Sure-thing logic says Y should not drive the ranking, yet preferences flip.

Why (I) & (IV) violates EU with one subjective probability

Normalise $u(0) = 0$ and let $u(100) > 0$.
Preferring (I) to (II) implies:

$$\mathbb{P}(R)\,u(100) > \mathbb{P}(B)\,u(100) \quad \Rightarrow \quad \mathbb{P}(R) > \mathbb{P}(B).$$

Preferring (IV) to (III) implies:

$$(\mathbb{P}(B) + \mathbb{P}(Y))u(100) > (\mathbb{P}(R) + \mathbb{P}(Y))u(100) \quad \Rightarrow \quad \mathbb{P}(B) > \mathbb{P}(R).$$

Contradiction. Hence EU with a single subjective probability measure cannot represent the pattern.

Interpretation: ambiguity aversion

Ellsberg choices suggest a preference for:

- gambles with **known** probabilities,
- over gambles with **unknown** probabilities,

even when payoffs are otherwise comparable.
Ambiguity aversion is not simply risk aversion:

- risk aversion: dislike variance with known probabilities,
- ambiguity aversion: dislike unknown probabilities (model uncertainty).

Real-world examples of ambiguity

Ambiguity often appears when probabilities are disputed or unstable:

- new technologies (unknown failure rates),
- emerging diseases (unknown base rates and transmission),
- rare disasters with changing climate conditions,
- new markets with limited data.

In such settings, "I do not know the probability" can be a key psychological driver.

Integrated learning activity 3: build an Ellsberg-style example

Activity (groups, 4 minutes). Create a real-world scenario with:

- one option that depends on a **known** probability,
- another option that depends on an **unknown** probability,
- identical payoffs.

Example domains: product failure risk, investment success, insurance claim acceptance, medical test accuracy.

Write the two options clearly and predict which one most people would choose.

Where we are

Why we need another model

Probability weighting explains Allais-type certainty effects, but Ellsberg is about **unknown** probabilities.

We need a model where:

- beliefs are not a single probability distribution,
- the decision-maker evaluates choices with **robustness** in mind.

A classic approach: **multiple priors** (maxmin expected utility).

Multiple priors: the formal idea

Let S be a set of states. Let $\mathcal{P}$ be a **set** of plausible probability measures over S. An act A yields outcome $x_A(s)$ in state s. With utility $u(\cdot)$, evaluate:

$$V(A) = \min_{P \in \mathcal{P}} \mathbb{E}_P\left[u(x_A(S))\right].$$

Decision rule:

$$A^\star \in \arg\max_A \min_{P \in \mathcal{P}} \mathbb{E}_P\left[u(x_A(S))\right].$$

Interpretation: choose the option with the best worst-case expected utility.

Intuition: an “ambiguity premium”

Under ambiguity, people may act as if:

- they subtract an extra penalty for not knowing p,
- or they focus on a pessimistic probability within a plausible range.

Multiple priors is one clean way to formalise this.
It is especially plausible in high-stakes settings where being wrong about probabilities is costly.

Worked example 9: Ellsberg with maxmin (Decision 1)

In the urn:

$$\mathbb{P}(R) = \frac{1}{3}, \qquad \mathbb{P}(B) = q, \qquad \mathbb{P}(Y) = \frac{2}{3} - q, \quad q \in \left[0, \frac{2}{3}\right].$$

Let $u(0) = 0$ and $u(100) = 1$.
Option (I): win if red:

$$V(I) = \min_{q \in [0,2/3]} \left(\frac{1}{3} \cdot 1\right) = \frac{1}{3}.$$

Option (II): win if black:

$$V(II) = \min_{q \in [0,2/3]} (q \cdot 1) = 0.$$

So maxmin predicts (I) over (II): prefer the known probability.

Worked example 10: Ellsberg with maxmin (Decision 2)

Option (III): win if red or yellow:

$$\mathbb{P}(R \cup Y) = 1 - q. \quad \Rightarrow \quad V(III) = \min_{q \in [0,2/3]} (1 - q) = \tfrac{1}{3}.$$

Option (IV): win if black or yellow:

$$\mathbb{P}(B \cup Y) = \tfrac{2}{3} \quad \Rightarrow \quad V(IV) = \tfrac{2}{3}.$$

So maxmin predicts (IV) over (III), matching the common Ellsberg pattern.

Worked example 11: robust project choice (AUD)

A project yields:

- profit A$ 500,000 if successful,
- loss A$ 200,000 if it fails.

The success probability is believed to be in $[0.2, 0.6]$. Let $\mathcal{P} = \{p : p \in [0.2, 0.6]\}$ and use $u(x) = x$.
Expected value at probability p:

$$\mathbb{E}[x] = p(500{,}000) + (1 - p)(-200{,}000) = 700{,}000p - 200{,}000.$$

Worst case occurs at $p = 0.2$:

$$\min \mathbb{E}[x] = 700{,}000(0.2) - 200{,}000 = -60{,}000.$$

Maxmin recommendation: do not invest (worst case is negative).

A softer version: α-maxmin (optional extension)

Some people are not fully pessimistic. A common extension combines worst and best cases:

$$V_\alpha(A) = \alpha \min_{P \in \mathcal{P}} \mathbb{E}_P[u(x_A)] + (1-\alpha) \max_{P \in \mathcal{P}} \mathbb{E}_P[u(x_A)],$$

with $\alpha \in [0,1]$.

- $\alpha = 1$: pure maxmin (very ambiguity averse),
- $\alpha = 0$: pure maxmax (very optimistic).

You should be able to interpret α as "how much weight is on the worst case".

Worked example 12: α-maxmin for the project

From the project:

$$\mathbb{E}[x] = 700{,}000p - 200{,}000, \quad p \in [0.2, 0.6].$$

Worst case: $-60{,}000$ at $p = 0.2$. Best case: $220{,}000$ at $p = 0.6$.
So:

$$V_\alpha = \alpha(-60{,}000) + (1-\alpha)(220{,}000) = 220{,}000 - 280{,}000\alpha.$$

Invest if $V_\alpha > 0$:

$$220{,}000 - 280{,}000\alpha > 0 \quad \Rightarrow \quad \alpha < \frac{220}{280} \approx 0.786.$$

If the decision-maker is sufficiently pessimistic ($\alpha \geq 0.786$), they avoid the project.

Integrated learning activity 4: ambiguity vs risk (calculation)

Activity (pairs, 4 minutes).
An investment returns:

- A\$ 100,000 with probability p,
- A\$ 0 with probability $1 - p$.

But p is ambiguous: $p \in [0.1, 0.4]$.

Use $u(x) = x$.

1. Compute the maxmin value of the investment.
2. Compute the maxmax value.
3. For α-maxmin, find the condition on α under which the investment is chosen, compared with not investing (value 0).

Where we are

Application 1: insurance pricing and uptake

Prospect theory and ambiguity aversion help explain:

- strong demand for low deductibles (loss aversion for out-of-pocket payments),
- sensitivity to rare events (probability weighting),
- distrust or avoidance when terms are unclear (ambiguity about exclusions and claims).

Managerial implication: transparency about probabilities and clear contract terms can change behaviour even if expected values do not.

Application 2: lotteries, gambling, and "lottery-like" products

Overweighting of small probabilities can produce:

- lottery ticket demand,
- attraction to very high-upside investments,
- willingness to accept bad average returns for a small chance of a large payoff.

Policy implication: consumer protection may need to consider **perceived** probabilities, not only objective odds.

Application 3: business decisions under model uncertainty

Firms often face ambiguous probabilities:

- new market entry (limited data),
- demand shocks (structural change),
- tail risks (supply-chain disruption).

Multiple priors reasoning captures why managers may:

- choose robust strategies,
- pay for information to reduce ambiguity,
- prefer flexibility (real options) when probability estimates are unstable.

Application 4: public communication of risk

In public health and disaster planning:

- small probabilities can be overweighted (panic) or underweighted (complacency), depending on context,
- framing outcomes as losses vs gains shifts preferences (reference dependence),
- ambiguity (uncertain evidence) can reduce compliance if not communicated well.

Good communication separates **risk** from **ambiguity** and provides ranges when appropriate.

Where we are

Common misconceptions

1. **"EU is wrong"** vs **"EU is a benchmark"**: EU is a useful baseline for diagnosis and policy design.
2. **Risk aversion** is not the same as **ambiguity aversion**.
3. **Probability weighting** does not mean probabilities are ignored; it means they are perceived non-linearly.
4. **Reference point** is not always "current wealth"; it can be an expectation, a target, or a norm.
5. **Certainty** is not just "a very high probability" in many people's psychological responses.

Pitfall checklist for problem solving

When you see a risky-choice question, ask:

1. Is this **risk** (known p) or **ambiguity** (unknown p)?
2. Under EU: what is $u(\cdot)$? Are we asked for EU, CE, or risk premium?
3. Under prospect theory: what is the **reference point** r? Are outcomes gains/losses?
4. Are probabilities being **weighted** (use $\pi(p)$) or taken as objective?
5. Under multiple priors: what is the set $\mathcal{P}$? Are we using min, max, or α-mix?

Where we are

Exam relevance: skills you should have

You should be able to:

- compute EU, CE, and risk premia for small lotteries,
- explain Allais and Ellsberg paradoxes clearly, including which axiom is violated,
- draw and interpret the prospect theory value function and probability weighting curve,
- compute simple prospect theory values given $v(\cdot)$ and $\pi(\cdot)$,
- solve maxmin and α-maxmin examples under ambiguity.

Practice MCQ 1

Question: The Allais paradox is most directly interpreted as a violation of:

1. transitivity
2. independence (sure-thing reasoning)
3. completeness
4. diminishing marginal utility

Practice MCQ 2

Question: The Ellsberg paradox is best described as evidence of:

1. ambiguity aversion (preference for known probabilities)
2. probability weighting only
3. risk neutrality
4. time inconsistency

Practice short-answer question

Question (6–8 sentences): Explain why the Ellsberg choices (I over II and IV over III) cannot be represented by EU with a single subjective probability. Your answer must:

- define what is meant by "unknown probability",
- use the sure-thing intuition (shared outcomes),
- include one inequality contradiction in simple words.

Practice long-answer question

Question (paragraphs + calculations):
A consumer is deciding whether to buy an extended warranty.

- Warranty premium: A$ 180.
- Without warranty: 4% chance of a repair cost of A$ 1,200.

1. Compute the expected value of buying vs not buying.
2. Explain how prospect theory (reference point, loss aversion, probability weighting) could increase warranty demand.
3. Explain how ambiguity aversion could affect the decision if the 4% probability is uncertain (e.g., "between 2% and 6%").

Where we are

1. Motivation and real-world relevance
2. Benchmark: expected utility and the independence idea
3. Allais paradox: the certainty effect
4. Probability weighting: why probabilities may not feel linear
5. Prospect theory: mechanics and intuition
6. Ellsberg paradox: ambiguity aversion
7. Multiple priors: a model of ambiguity aversion
8. Applications: consumer, business, and policy decisions
9. Common misconceptions and pitfalls
10. Exam preparation
11. **Key takeaways and next week**

Key takeaways

1. EU is a powerful benchmark, but it relies on independence/sure-thing logic.
2. The Allais paradox highlights the **certainty effect**: probability 1 is psychologically special.
3. Probability weighting captures why small probabilities and near certainty can feel non-linear.
4. Prospect theory combines reference dependence, loss aversion, and probability weighting to explain systematic patterns.
5. The Ellsberg paradox highlights **ambiguity aversion**, which can be modelled via multiple priors (maxmin).

Preview of next week

Next week moves from uncertainty about outcomes to uncertainty over **time**:

- intertemporal choice and discounting,
- present bias and dynamic inconsistency,
- applications to saving, debt, health behaviour, and policy design.

Bring your "benchmark vs behaviour" mindset: start with a clean model, then diagnose systematic deviations.

Where we are

Answer: Activity 1 (Allais reasoning guide)

Key points a good answer should include:

- In Decision 1, many prefer the sure A$ 1M because certainty is valued disproportionately.
- Independence says the common 0.89 state paying A$ 1M in both (1A) and (1B) should not affect ranking.
- In Decision 2 there is no certainty; many focus on the bigger prize and choose (2B).
- The combination (1A) and (2B) cannot be represented by EU because it implies contradictory inequalities.

Answer: Activity 2 (prospect values ranking)

Given $v(\Delta) = \sqrt{\Delta}$ and $v(0) = 0$:

$$V(A) = \sqrt{90} \approx 9.49.$$

$$V(B) = \pi(0.10)\sqrt{1000} = 0.18 \cdot 31.62 \approx 5.69.$$

$$V(C) = \pi(0.90)\sqrt{120} = 0.80 \cdot 10.95 \approx 8.76.$$

Ranking: A (9.49) $>$ C (8.76) $>$ B (5.69).

Answer: Activity 3 (example structure)

A correct Ellsberg-style example must have:

- a known probability option (e.g., "a regulated product with audited failure rate 2%"),
- an unknown probability option (e.g., "a new product with unknown failure rate"),
- identical payoffs.

Predicted pattern: many choose the known-probability option, revealing ambiguity aversion.

Answer: Activity 4 (maxmin and α-maxmin)

Payoff is A\$ 100,000 with probability $p \in [0.1, 0.4]$, otherwise 0. With $u(x) = x$:

$$\mathbb{E}[x] = 100{,}000\, p.$$

Maxmin (worst case at $p = 0.1$):

$$V_{\text{min}} = 10{,}000.$$

Maxmax (best case at $p = 0.4$):

$$V_{\text{max}} = 40{,}000.$$

α-maxmin:

$$V_\alpha = \alpha(10{,}000) + (1 - \alpha)(40{,}000) = 40{,}000 - 30{,}000\alpha.$$

Compared with not investing (value 0), invest for all $\alpha \in [0, 1]$ because $V_\alpha > 0$ always.

Answers: Practice MCQs

MCQ 1: (2) independence (sure-thing reasoning).

MCQ 2: (1) ambiguity aversion.